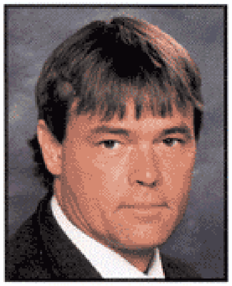
Estimating vibration at critical locations using available measurements and machine configuration data



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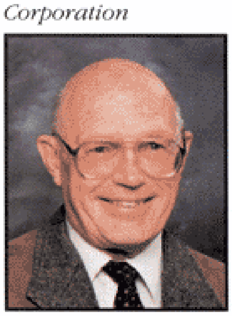

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Nomenclature

As	= Element shaft cross sectional area
Ad	= Disk cross sectional area
Db	= Bearing damping
Ii	= Element shaft inside diameter bending moment of inertia
Io	= Element shaft outside diameter bending moment of inertia
Kb	= Bearing stiffness
Kd	= Bearing direct dynamic stiffness
Kq	= Bearing quadrature dynamic stiffness
Ld	= Disk length
Ls	= Element shaft length
SNR	= Signal to noise ratio
λ	= Average circumferential fluid velocity
pd	= Disk density
ps	= Element shaft density
ω	= Forcing function rotative speed
Ω	= Rotor rotative speed

One of the problems associated with establishing acceptable vibration levels for rotating machinery is noncollocation of critical vibration and available measurement locations. Noncollocation of measurement and critical machinery monitoring locations has been a problem faced by machinery engineers since the first measurement probes were installed on machinery. Critical machinery monitoring locations are any location along the rotor with a reduced clearance or a high potential for rotor to stator contact.

It is difficult to estimate what the measured vibration at the bearings will be just before the available clearance is exceeded at an internal location. Unknown forcing functions, operation through several lateral modes, and scarcity of measurement locations complicate the estimation process. For rigid machines, these problems are easily overcome. However, for flexible machines, which must pass through several lateral modes getting to operating speed, this becomes more difficult. The difficulty increases because the correlation between measured and critical vibration responses changes with imbalance distribution, operating speed, natural rotor modes, and other operating conditions. The creation of two measurement locations near the bearings, namely, "modal" probes, instead of just one, increases the measurement location density, which can lead to an improvement in the estimation of the actual operating mode. If a multi-bearing machine train (Figure 1) is considered, several data interpolation/extrapolation techniques could be used, some of which are outlined below.

Linear Extrapolation: For linear extrapolation, the bearing closest to the desired location is selected. The equation of the straight line through the two data points on either side of the bearing can then be used to find the response amplitude and phase at other axial locations. This technique is useful for rigid modes, but starts producing errors as soon as the rotor begins to bend. Clearly, another technique must be found if extrapolation for rotor bending modes is required.

Polynomial Extrapolation: Polynomial extrapolation requires that a polynomial of order one less than the►

number of measurement locations be generated. This polynomial equation can then be used to calculate the vibration response at any axial location. This technique might seem to be ideal for the task at hand. However, high order interpolation polynomials tend to produce large errors for a great many functions, especially if the measured signals are contaminated by noise.

Spline Fits: The mathematician's answer to the errors introduced by high order polynomials are spline fits. Generally, spline fits replace a single high order polynomial with several lower order polynomials, each of which is used for some portion of the data range. To smooth the transitions from one polynomial to the next, both the response amplitude and phase, and several of its derivatives, must be identical to adjoining polynomials at the transition points. The measurements give the response vectors, but there are no measurement devices that measure derivatives directly. Therefore, they must be calculated from the response amplitudes. Modal probes allow the first derivative to be approximated reasonably well. However, to calculate the second derivative, another point is necessary. A spline fit algorithm would go to the next available data point to get this required information. However, if we notice how far that point is from the desired location, across a rotor span, you have to wonder whether this point provides any useful information about the second derivative at the original location. Because of this, spline fits are not good candidates for extrapolation of responses along machine trains.

Cubic Polynomials: If each rotor span is processed separately, a cubic polynomial could be constructed which passes through the four measurement locations at the support bearings. This allows good approximations up to and including the second bending mode for the selected rotor span. Assuming that other, more relevant, information is not available, this technique probably gives the best results in the field.

However, some additional information exists which could lead to better extrapolated responses. The physical configuration of the system is useful data that is often overlooked. This includes the bearing locations and their stiffnesses, the operating speed and the physical description of the rotor. There are several methods suitable for using system information and measured data to extrapolate vibration responses. Eigenvectors can be computed and used as a new modal coordinate system. The number of degrees of freedom can then be reduced by discarding all coordinates except those that describe the desired modes. Either optimization or autocorrelation techniques can be used to determine the coefficients of each mode. The response at the desired location can then be obtained by translating back to the original coordinate system.

An alternate way to compute vibration responses at the measured locations is to optimize the imbalance force distribution to minimize the error between these computed responses and the actual measured responses. This alternate approach is based on finite element or transfer matrix techniques. Similar techniques have been used for machinery parameter identification by other researchers (Nordmann, Diewald (1990), Mottershead (1991), Muszynska, et al (1992)). A computer program implementing this process, along with preliminary results from experimental testing, is presented.

Finite element optimization technique

The flow diagram for the computer program is shown in Figure 2. The machinery engineer must first input the system configuration. This includes dividing the rotor into finite elements, identifying the bearing locations and stiffnesses, the measurement locations, the critical or extrapolation locations and the imbalance forces. The only constraint in dividing the rotor into finite elements is that measurement and critical locations must occur at the left end of an element. For good modeling accuracy, the rotor must be broken into enough elements for adequate representation. This usually results in more computational nodes than desired extrapolation points. To specify which computational nodes correspond with extrapolation locations, the concept of virtual measurement locations is used.

Measurement locations can be declared as either real or virtual. Real measurement locations get their data directly from the data acquisition system. Virtual locations receive their data from the program's computed response for the node corresponding with that location, once the optimization is complete. The computed data at other computational nodes is discarded.

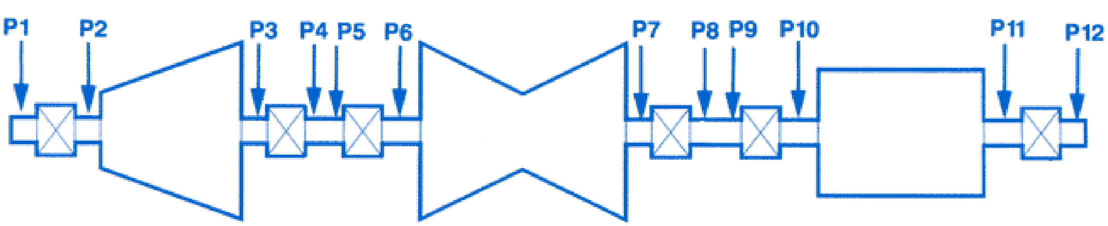

Figure 1
Typical modal probe installation on a multi-bearing machine.

This technique reduces the amount of memory required for data storage, and also formats the extrapolated data as if it were measured data, thereby allowing the use of the same presentation routines for both types of data. If all the system parameters — rotor configuration, support stiffnesses, operating speed, and imbalance force distribution — are known, the responses at the extrapolation locations can be computed directly. However, this is usually not the case. The rotor parameters (dimensions and materials of the shaft and attached disks) can be obtained from drawings or measurements. The bearing stiffnesses are a little harder to obtain since they depend on installation and operating conditions. However, reasonably good approximations may be obtained using perturbation or other techniques. The use of this program, in conjunction with synchronous perturbation techniques to determine bearing stiffnesses, is discussed by Muszynska, et al (1992).

If the imbalance force distribution is known, the responses at the critical locations can be computed directly without any optimization. However, this is rarely the case. The goal of the optimization technique is to get a good enough approximation to the actual imbalance force distribution that it can be used to compute the responses at the critical locations. Any known imbalance forces can be input. However, imbalance forces specified at this time for locations which will be defined as optimization locations will only be used to compute initial conditions.

The second task shown in the flow diagram is to determine which elements contain imbalance forces that will be involved in the optimization process. Allowing control over the number of optimization variables lets the user trade length of time to get a solution with the resolution of the approximation to the imbalance distribution. At this point in the program, all of the initial data available to the user has been input to the program. The program also uses transient machinery data captured by a data acquisition system. For brevity, the optimization process will be explained at only one speed, but it is actually performed at each sampled rotative speed contained in the data.

The first step in the optimization loop is to obtain the measured data that will be used as the reference data. Initial values are then assigned to the imbalance forces selected as optimization variables. The next step is to compute the system stiffness matrix.

To build an algorithm that can operate efficiently on a personal computer, some limitations and special techniques have been implemented in the construction of both the element and system stiffness matrices. The modeling elements must be axisymmetric cylindrical elements or external disks attached to the shaft elements with no gyroscopic effects, and rigid foundations at the support locations are assumed. These limitations are not inherent to the finite element calculation technique. They have been imposed on this program to drastically reduce the computational time needed to produce the system response to a particular imbalance distribution without significantly affecting the accuracy of the results for the rotor system used as a test case. This produces a much more efficient environment for research into optimization techniques, the primary research subject of this paper, since results from changes to the program algorithms can be obtained faster.

Once the optimization algorithms have been finalized, the limitations in the finite element modeling can be removed, allowing more complex rotor systems to be accurately modeled. These limitations do make the current program inadequate for some machines. However, it is still applicable to a large number of rotating machines currently in service. This is especially true if the computed response is allowed the same magnitude of error usually inherent in the measurement process. These limitations force the system stiffness matrix to be the narrowest band symmetric formulation possible, which not only allows the use of the simplest and fastest matrix inversion algorithms but also reduces the amount of computer memory required to store the matrix. This is an important consideration when using personal computers with DOS operating systems.

In addition to element modeling limitations, superposition is used to reduce computational time. Each element stiffness matrix is constructed in layers (Figure 3). The base layer contains all of ►

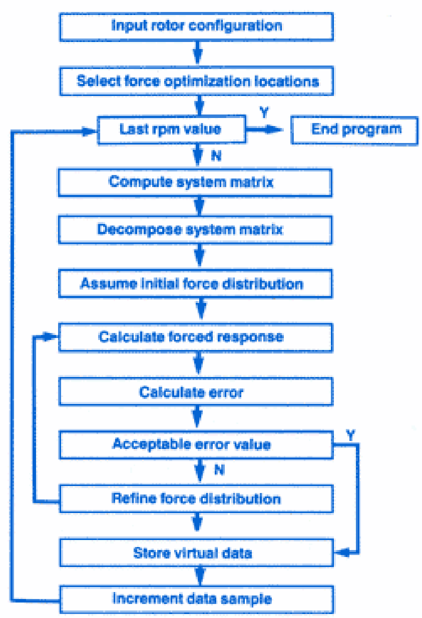

Figure 2
Flow diagram for computer program to extrapolate measurements to other axial locations.

the stiffness terms for the element that do not contain optimization variables. If the element is not selected as an optimization location, the base layer contains all three layers, mass, shaft stiffness, and bearing stiffness, and the optimization layer is empty.

If the shaft stiffness of the element is selected as an optimization parameter, then the shaft stiffness layer is placed in the optimization layer instead of the base layer. The same is true for the bearing stiffness layer if bearing stiffness is selected as an optimization parameter. The mass layer is always placed in the base layer since mass can not be selected as an optimization parameter. The system matrix is also constructed in layers, a base layer and an optimization layer (Figure 4).

The system base layer is generated using the element base layers, while the system optimization layer is generated from the element optimization layers. This speeds computation, since only the terms associated with the optimization variables need to be recomputed for each iteration, instead of the whole system matrix. Note that the imbalance force matrix is constructed in layers in the same manner as the system stiffness matrix. Once the force and stiffness matrices exist, the response matrix can be solved.

Note that, for data extrapolation, all of the optimization variables are contained in the force matrix. If the system matrix in a solution technique can be modified once and then used in its modified form for future calculations, a significant reduction in computational time will result. This can be accomplished with either full inversion or LU decomposition. This program uses a variation of LU decomposition, the square root method (Al-Khafaji, et al, 1986), which takes into account the banded symmetric diagonal form of the stiffness matrix to reduce the computation necessary to produce the decomposed form.

After the response has been calculated, the least squared error is computed between the measured and calculated data. If the error is acceptably small, the required calculated responses are placed in the virtual measurement locations and the program proceeds to the next rotative speed. If the error is too large, the program uses Powell's method of discarding the direction of largest decrease (Press, et al, 1986) to control the optimization directions and Brent's method (Press, et al, 1986) for finding the minimum along a direction to refine the force distribution. A new response matrix is computed using the new force distribution, and the error recomputed. This process is repeated until the error between the calculated and measured responses is acceptably small.

Experimental results

To test the validity of the technique and the program, a small rotating

$$\underbrace{M\begin{bmatrix} -156 & -22Ls & -54 & 13Ls \\ -22Ls & -4Ls^2 & -13Ls & 3Ls^2 \\ -54 & -13Ls & -156 & 22Ls \\ 13Ls & 3Ls^2 & 22Ls & -4Ls^2 \end{bmatrix}}_{\text{element stiffness matrix mass layer}} + \underbrace{S\begin{bmatrix} 12 & 6Ls & -12 & 6Ls \\ 6Ls & 4Ls^2 & -6Ls & 2Ls^2 \\ -12 & -6Ls & 12 & -6Ls \\ 6Ls & 2Ls^2 & -6Ls & 4Ls^2 \end{bmatrix}}_{\text{element stiffness matrix shaft stiffness layer}} + \underbrace{\begin{bmatrix} Kd+jKq & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & Kd+jKq & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{element stiffness matrix bearing stiffness layer}} = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{\text{element stiffness matrix base layer}} + \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{\text{element stiffness matrix optimization layer}}$$

$$M = \frac{(\rho sLsAs + \rho dLdAd)}{420} \qquad S = \frac{(Io - Ii)E}{16Ls^3} \qquad Kd = \frac{Kb}{2} \qquad Kq = \frac{Db(\omega - \lambda\Omega)}{2}$$

Figure 3
Layered construction of element stiffness matrix.

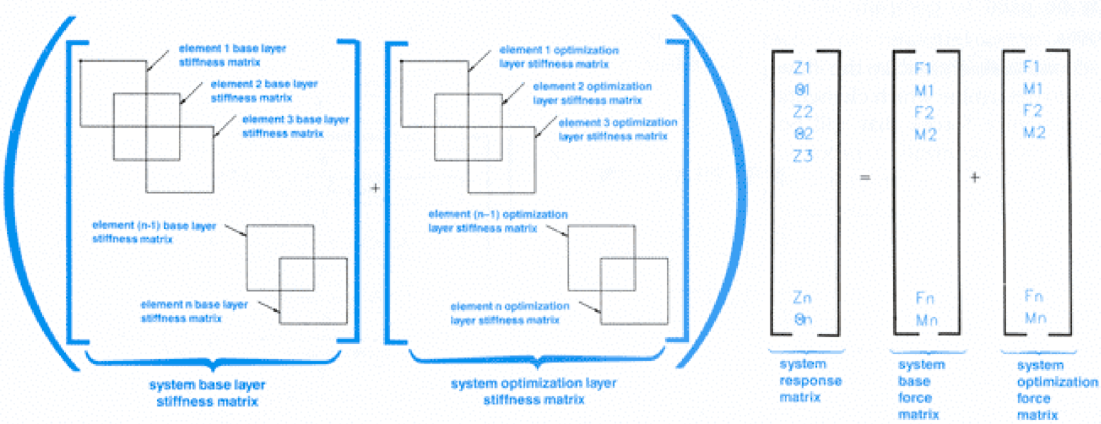

Figure 4
System forced response equations, showing layered construction of stiffness and force matrices.

machine was constructed using oilite bearings and a 0.375 inch (9.5 mm) diameter shaft 18 inches (457 mm) long with two heavy discs attached. The system was driven by a 0.1 hp (75 watt) motor through a flexible coupling. Synchronous measurements were taken with an eight-channel data acquisition/ filtering system and then downloaded to the personal computer for postprocessing. Figure 5 is a block diagram of the system. The rotor was modeled by twenty-six elements (Figure 6).

Notice that each probe location has a corresponding computational node which can be directly compared with the measured value during the optimization process. The bearing stiffness values were determined using synchronous force perturbation methodology and this program with the bearing stiffness parameters as the optimization variables. Once these values were determined, they were assumed to be constant, which fixed all the terms in the system stiffness matrix. At this point, a startup was performed and data was collected from all of the measurement locations. Different sets of modal probes were selected at each end of the rotor, and the responses at the other measured locations were then extrapolated and compared with the actual measured responses. If one of the modal probes is located near an antinodal point, the extrapolated and measured responses are virtually identical.

As both the modal probes approach a nodal point, the error in the extrapolated responses tends to increase (Figure 7), but is still within 20 percent, an acceptable range for machinery diagnostic work. The noise or uncertainty of the measurements seems to be fairly constant along the shaft, which produces a good signal-to-noise ratio (SNR) near the antinodal points and a poorer SNR near the nodal points. The quality of the extrapolation is directly proportional to the SNR. This indicates that the technique is highly accurate as long as good SNRs are maintained in the measured data. The computed force distribution required to optimize the error term is almost identical with the imbalance force used to generate the data (Figure 8). This indicates that the method might enhance balancing techniques, requiring no calibration weight runs to get the imbalance distribution. The influence vectors are effectively calculated from the system stiffness matrix, so manual calibration runs can be eliminated.

Final remarks

The technique of extrapolating vibration response from measured planes to other axial locations on the rotating machine, using finite element modeling, optimization theory and actual measured responses, provides useful information for machinery protection and diagnostics. It works acceptably well for the laboratory experiments performed►

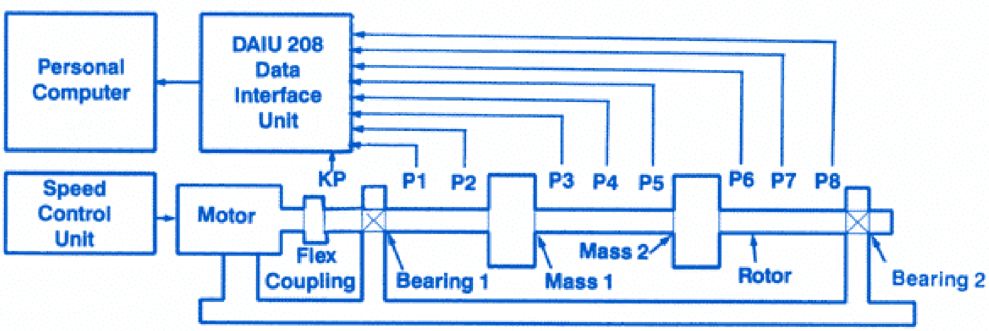

Figure 5
Block diagram of experimental rotor rig and instrumentation used to verify operation of computer program to extrapolate vibration response.

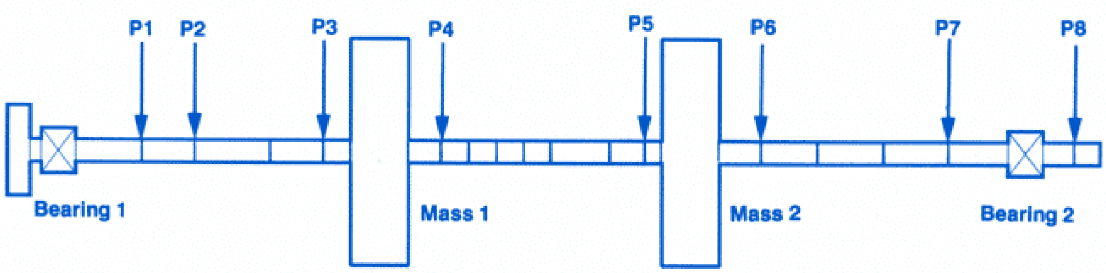

Figure 6
Experimental rotor rig shaft showing finite element construction and measurement transducer locations.

to evaluate whether the technique would work under controlled conditions. These experiments were not designed to be field representative. More work must be done to validate the procedure for field use.

However, the preliminary results are encouraging. If good SNRs can be maintained in the field, the technique will probably produce acceptable results. Currently, measurements tend to be taken at the bearings, which are usually near nodal points. This practice doesn't produce optimal measured data. However, the greater density of measurements, when modal probes are used, increases the database and improves the technique's performance. Additional data processing and different optimization algorithms may improve results even further. Both items are currently under investigation. Computational complexity can be reduced to give acceptable performance on personal computers. As computational power is increased, either speed or modeling complexity can be increased. In short, the procedure can enhance future diagnostic systems, but further research into error terms and SNR increases may be necessary to allow the technique to function adequately for inexperienced users in field applications.■

This is a subject that I am very interested in. I would appreciate hearing your thoughts or experiences on this subject. I would be grateful for any information you can send me, including actual machine data and case histories. I will keep you informed of the progress of my work.

Donald E. Bently

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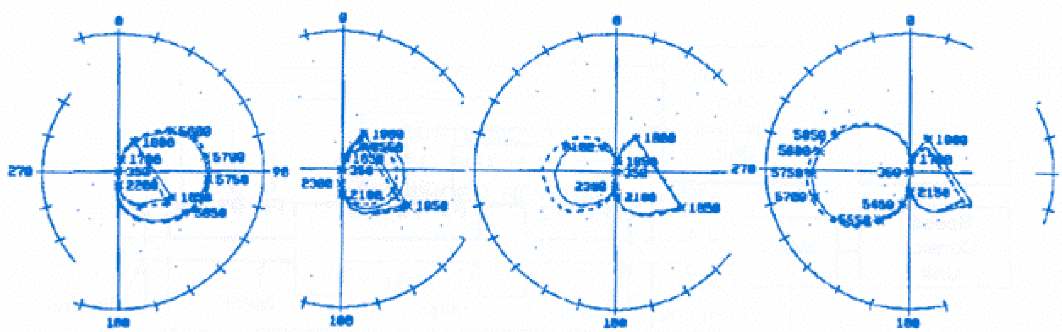

Figure 7

Extrapolated data for measurement planes 3, 4, 5, and 6 using measurement planes 1, 2, 7, and 8 as modal measurement locations, presented in the form of Polar plots of synchronous vibration. Solid lines are measured data, broken lines are extrapolated data.

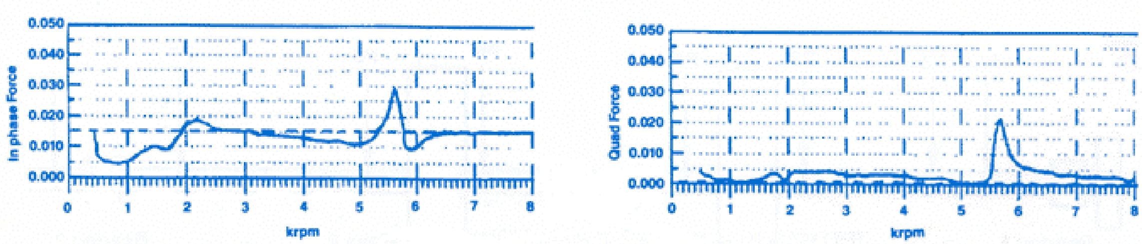

Figure 8

Imbalance force distribution versus rotative speed, created during optimization process. Solid lines are computed responses. Broken lines are the imbalance forces used to create the transient data. The left plot is the direct component of the imbalance force, while the right is the quadrature component. "Direct" and "quadrature" refer here to orthogonal coordinates referenced to the measurement transducers.